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CURSO DE PÓS - GRADUAÇÃO LATO SENSU MATEMÁTICA E ESTATISTICA - ÁLGEBRA DE MATRIZES

CORUMBIARA-RO, 29/04/06

Curso Pós-Graduação *Lato Sensu* Matemática e Estatística - Álgebra de Matrizes
21/03/2006 Professor: Lucas Monteiro Chaves

1) Resolva de três modos diferentes o sistema $\begin{cases} x_1 + x_2 + x_3 + x_4 = 4 \\ x_1 - x_2 + x_3 + x_4 = 8 \\ x_1 + x_2 - 2x_3 - 2x_4 = -11 \\ 2x_1 + x_2 + 2x_3 + x_4 = 1 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & -2 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ -11 \\ 1 \end{bmatrix}$ Primeiro modo resolução pela Fatoração LU

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & -2 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{matrix} \\ -L_1 + L_2 \\ -L_1 + L_3 \\ -2L_1 + L_4 \end{matrix} \Longrightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & -3 \\ 0 & -1 & 0 & -1 \end{bmatrix} \text{-}\tfrac{1}{2}L_2 + L_4 \Longrightarrow$$

$$\Longrightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$m_{21} = 1 \qquad m_{31} = 1 \qquad m_{41} = 2 \qquad m_{32} = 0 \qquad m_{42} = \frac{1}{2}$

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & \frac{1}{2} & 0 & 1 \end{bmatrix}$$

$Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & \frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ -11 \\ 1 \end{bmatrix}$$

$y_1 = 4 \qquad y_1 + y_2 = 8 \qquad y_1 + y_3 = -11 \qquad 2y_1 + \frac{1}{2}y_2 + y_4 = 1$

$y_2 = 4 \qquad y_3 = -11 - 4 \qquad y_4 = 1 - 8 - 2$

$y_3 = -15 \qquad y_4 = -9$

$Ux = y$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ -15 \\ -9 \end{bmatrix}$$

$-x_4 = -9 \qquad -3x_3 - 3x_4 = -15 \qquad -2x_2 = 4 \qquad x_1 + x_2 + x_3 + x_4 = 4$

$x_4 = 9 \qquad -3x_3 = -15 + 27 \qquad x_2 = -2 \qquad x_1 - 2 - 4 + 9 = 4$

$x_3 = \frac{12}{-3} \qquad x_1 = 4 + 4 + 2 - 9$

$x_3 = -4 \qquad x_1 = 1$

Logo temos uma solução que é $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -4 \\ 9 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & -2 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ -11 \\ 1 \end{bmatrix}$ Nesta segunda resolução utilizarei o Teorema de Rouché-Capelli

$$\begin{bmatrix} 1 & 1 & 1 & 1 & : & 4 \\ 1 & -1 & 1 & 1 & : & 8 \\ 1 & 1 & -2 & -2 & : & -11 \\ 2 & 1 & 2 & 1 & : & 1 \end{bmatrix} \begin{matrix} \\ -L_1 + L_2 \\ -L_1 + L_3 \\ -2L_1 + L_4 \end{matrix} \Longrightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & : & 4 \\ 0 & 2 & 0 & 0 & : & 4 \\ 0 & 0 & -3 & -3 & : & -15 \\ 0 & -1 & 0 & -1 & : & -7 \end{bmatrix} \begin{matrix} \\ -\frac{1}{2}L_2 \\ -\frac{1}{3}L_3 \\ -\frac{1}{2}L_2 + L_4 \end{matrix} \Longrightarrow$$

$$\Longrightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & : & 4 \\ 0 & 1 & 0 & 0 & : & -2 \\ 0 & 0 & 1 & 1 & : & 5 \\ 0 & 0 & 0 & -1 & : & -9 \end{bmatrix} \begin{matrix} L_1 + L_2 \\ \\ L_3 + L_4 \\ (-1)\,L_4 \end{matrix} \Longrightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & : & 6 \\ 0 & 1 & 0 & 0 & : & -2 \\ 0 & 0 & 1 & 0 & : & -4 \\ 0 & 0 & 0 & 1 & : & 9 \end{bmatrix} L_1 - L_3 \Longrightarrow$$

$$\Longrightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & : & 10 \\ 0 & 1 & 0 & 0 & : & -2 \\ 0 & 0 & 1 & 0 & : & -4 \\ 0 & 0 & 0 & 1 & : & 9 \end{bmatrix} L_1 - L_4 \Longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & 0 & : & -2 \\ 0 & 0 & 1 & 0 & : & -4 \\ 0 & 0 & 0 & 1 & : & 9 \end{bmatrix}$$

Tendo como solução de segundo modo de resolução $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -4 \\ 9 \end{bmatrix}$

No terceiro modo de resolver o sistema utilizarei o Método da Matriz Inversa

$Ax = b \Longrightarrow x = A^{-1}.b$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & -2 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ -11 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cccccccccc} 1 & 1 & 1 & 1 & : & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & : & 0 & 1 & 0 & 0 \\ 1 & 1 & -2 & -2 & : & 0 & 0 & 1 & 0 \\ 2 & 1 & 2 & 1 & : & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -L_1 + L_2 \\ -L_1 + L_3 \\ -2L_1 + L_4 \end{array} \Longrightarrow$$

$$\Longrightarrow \left[\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & : & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & : & -1 & 1 & 0 & 0 \\ 0 & 0 & -3 & -3 & : & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & : & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} L_1 + L_4 \\ -\frac{1}{2}(L_2) \\ -\frac{1}{3}(L_3) \\ \end{array} \Longrightarrow$$

$$\left[\begin{array}{ccccccccc} 1 & 0 & 1 & 0 & : & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 1 & : & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & -1 & 0 & -1 & : & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ L_4 + L_2 \end{array} \Longrightarrow$$

$$\Longrightarrow \left[\begin{array}{ccccccccc} 1 & 0 & 1 & 0 & : & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 1 & : & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 & : & -\frac{3}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right] \begin{array}{l} -L_3 + L_1 \\ \\ L_3 + L_4 \\ (-1)\,L_4 \end{array} \Longrightarrow$$

$$\Longrightarrow \left[\begin{array}{ccccccccc} 1 & 0 & 0 & 0 & : & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 1 & 0 & 0 & : & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 1 & : & -\frac{7}{6} & -\frac{1}{2} & -\frac{1}{3} & 1 \\ 0 & 0 & 0 & 1 & : & \frac{3}{2} & \frac{1}{2} & 0 & -1 \end{array}\right] \Longrightarrow$$

$$\begin{bmatrix} \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ -\frac{7}{6} & -\frac{1}{2} & -\frac{1}{3} & 1 \\ \frac{3}{2} & \frac{1}{2} & 0 & -1 \end{bmatrix} . \begin{bmatrix} 4 \\ 8 \\ -11 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$x_1 = \frac{2}{3} + 4 - \frac{11}{3}$ $\quad x_2 = 2 - 4$ $\quad x_3 = \frac{-14}{3} - 4 + \frac{11}{3} + 1$ $\quad x_4 = 6 + 4 - 1$

$x_1 = \frac{2+12-11}{3}$ $\quad x_2 = -2$ $\quad x_3 = \frac{-14-12+11+3}{3}$ $\quad x_4 = 9$

$x_1 = \frac{3}{3}$ $\quad x_3 = \frac{-12}{4}$

$x_1 = 1$ $\quad x_3 = -4$

Sendo a solução $s = \{1; -2; -4; 9\}$

2) Calcule o posto da matriz $\begin{pmatrix} 1 & 1 & 1 & -1 \\ 5 & 2 & 0 & 1 \\ -1 & 3 & 0 & 5 \\ 1 & 0 & 1 & 1 \\ 4 & 0 & 1 & 1 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 5 & 2 & 0 & 1 \\ -1 & 3 & 0 & 5 \\ 1 & 0 & 1 & 1 \\ 4 & 0 & 1 & 1 \end{pmatrix}$$, rank: 4, calculado pelo Algorítimo de Dwivedi como segue.

Seja A, $a_{11} = 1$

$$U_1 = \frac{1}{1}\begin{bmatrix} 1 \\ 5 \\ -1 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \\ 1 \\ 4 \end{bmatrix}; \qquad V\acute{}_1 = \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix}$$

$$U_1V\acute{}_1 = \begin{bmatrix} 1 \\ 5 \\ -1 \\ 1 \\ 4 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 5 & 5 & 5 & -5 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 4 & 4 & 4 & -4 \end{bmatrix}$$

$$A_1 = A - U_1V\acute{}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -3 & -5 & 6 \\ 0 & 4 & 1 & 4 \\ 0 & -1 & 0 & 2 \\ 0 & -4 & -3 & 5 \end{bmatrix} \neq \varphi$$

Seja em $A_1, a_{22} = -3$

$$U_2 = -\frac{1}{3}\begin{bmatrix} 0 \\ -3 \\ 4 \\ -1 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -\frac{4}{3} \\ \frac{1}{3} \\ \frac{4}{3} \end{bmatrix} \qquad V_2 = \begin{bmatrix} 0 & -3 & -5 & 6 \end{bmatrix}$$

$$U_2V\acute{}_2 = \begin{bmatrix} 0 \\ 1 \\ -\frac{4}{3} \\ \frac{1}{3} \\ \frac{4}{3} \end{bmatrix}\begin{bmatrix} 0 & -3 & -5 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -3 & -5 & 6 \\ 0 & 4 & \frac{20}{3} & -8 \\ 0 & -1 & -\frac{5}{3} & 2 \\ 0 & -4 & -\frac{20}{3} & 8 \end{bmatrix}$$

$$A_2 = A_1 - U_2V\acute{}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{17}{3} & 12 \\ 0 & 0 & \frac{5}{3} & 0 \\ 0 & 0 & \frac{11}{3} & -3 \end{bmatrix} \neq \varphi$$

Seja em $A_2, a_{33} = -\frac{17}{3}$

$$U_3 = -\frac{1}{\frac{17}{3}}\begin{bmatrix} 0 \\ 0 \\ -\frac{17}{3} \\ \frac{5}{3} \\ \frac{11}{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -\frac{5}{17} \\ -\frac{11}{17} \end{bmatrix}; \qquad V\acute{}_3 = \begin{bmatrix} 0 & 0 & -\frac{17}{3} & 12 \end{bmatrix}$$

$$U_3V'_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -\frac{5}{17} \\ -\frac{11}{17} \end{bmatrix} \begin{bmatrix} 0 & 0 & -\frac{17}{3} & 12 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{17}{3} & 12 \\ 0 & 0 & \frac{5}{3} & -\frac{60}{17} \\ 0 & 0 & \frac{11}{3} & -\frac{132}{17} \end{bmatrix}$$

$$A_3 = A_2 - U_3V'_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{60}{17} \\ 0 & 0 & 0 & \frac{81}{17} \end{bmatrix} \neq \varphi$$

Seja em $A_3, a_{44} = \frac{60}{17}$

$$U_4 = \frac{17}{60} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{60}{17} \\ \frac{81}{17} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \frac{27}{20} \end{bmatrix}; \qquad V'_4 = \begin{bmatrix} 0 & 0 & 0 & \frac{60}{17} \end{bmatrix}$$

$$U_4V'_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \frac{27}{20} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & \frac{60}{17} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{60}{17} \\ 0 & 0 & 0 & \frac{81}{17} \end{bmatrix}$$

$$A_4 = A_3 - U_4V'_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \varphi$$

Pelo Algorítimo de Dwivedi, a matriz A pode ser fatorada na forma $A = BC$

Com $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ -1 & -\frac{4}{3} & 1 & 0 \\ 1 & \frac{1}{3} & -\frac{5}{17} & 1 \\ 4 & \frac{4}{3} & -\frac{11}{17} & \frac{27}{20} \end{bmatrix}$ e $C = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & -3 & -5 & 6 \\ 0 & 0 & -\frac{17}{3} & 12 \\ 0 & 0 & 0 & \frac{60}{17} \end{bmatrix}$

Sendo que $A_r = \varphi$ e $A_4 = \varphi$ pode-se afirmar que o posto da matriz a seja 4.

3) Determine se os conjuntos de vetores são LD ou LI

a) $(2,3,4), (1,1,-1), (1,2,1)$

$x(2,3,4) + y(1,1,-1) + z(1,2,1) = (0,0,0,0)$

$(2x + y + z, 3x + y + 2z, 4x - y + z) = (0,0,0,0)$

$$\begin{cases} 2x + y + z = 0 \\ 3x + y + 2z = 0 \\ 4x - y + z = 0 \end{cases} \sim \begin{cases} 2x + y + z = 0 \\ -\frac{1}{2}y + \frac{1}{2}z = 0 \\ -3y - z = 0 \end{cases} \sim \begin{cases} 2x + y + z = 0 \\ -\frac{1}{2}y + \frac{1}{2}z = 0 \\ -4z = 0 \end{cases}$$

Dai a solução é única a trivial, e o conjunto é linearmente independente.

Sendo a solução $\{y = 0, x = 0, z = 0\}$

b) $(0,1,1,0), (2,0,0,2), (1,2,2,1)$

$x(0,1,1,0) + y(2,0,0,2) + z(1,2,2,1) = (0,0,0,0)$

$(2y+z, x+2z, x+2z, 2y+z) = (0,0,0,0)$

$$\begin{cases} 2y+z=0 \\ x+2z=0 \\ x+2z=0 \\ 2y+z=0 \end{cases} \sim \begin{cases} x+2z=0 \\ 2y+z=0 \end{cases}.$$

Esse sistema admite outras soluções além da trivial, dai o conjunto é linermente dependente

c) $(1,1,1,1), (0,2,0,2), (3,4,5,6), (1,1,6,3), (1,2,2,5)$

$$t(1,1,1,1)+u(0,2,0,2)+v(3,4,5,6)+x(1,1,6,3)+z(1,2,2,5) = (0,0,0,0) \begin{cases} t+3v+x+z=0 \\ t+2u+4v+x+2z=0 \\ t+5v+6x+2z=0 \\ t+2u+6v+3x+5z=0 \end{cases} \sim$$

$$\begin{cases} t+3v+x+z=0 \\ t+2u+4v+x+2z=0 \\ 2v+5x+z=0 \\ 2v+2x+3z=0 \end{cases} \sim \begin{cases} t+2u+4v+x+2z=0 \\ t+3v+x+z=0 \\ 2v+5x+z=0 \\ -3x+2z=0 \end{cases} \sim\sim \begin{cases} t+2u+4v+x+2z=0 \\ -2u-v-z=0 \\ 2v+5x+z=0 \\ 3x+2z=0 \end{cases}$$

Este sistema admite outras soluções além da trivial; dai o conjunto de vetores é linearmente dependente.

4) Utilize o Algorítimo de Dawivedi para fatorar a matriz $\begin{pmatrix} 1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 2 \\ 0 & 2 & 2 & 2 \\ 1 & 0 & 0 & -1 \end{pmatrix}$,

rank: 3

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 2 \\ 0 & 2 & 2 & 2 \\ 1 & 0 & 0 & -1 \end{pmatrix} \qquad U_1 = \frac{1}{pq}\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{mq} \end{bmatrix}; \qquad V\acute{}_1 =$$

$[a_{p1}, a_{p2}, ..., a_{pn}]$

$a_{11} = 1$

$$U_1 = \frac{1}{1}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \qquad V\acute{}_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\therefore U_1 V\acute{}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$A_1 = A - U_1 V\acute{}_1 =$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 2 \\ 0 & 2 & 2 & 2 \\ 1 & 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

Como $A_1 \neq \varphi$ repetir o processo em A_1

Seja em A_1, $a_{22} = 2$

obter $U_2V\acute{}_2$

$$U_2 = \frac{1}{2}\begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}; \qquad V\acute{}_2 = \begin{bmatrix} 0 & 2 & 2 & 2 \end{bmatrix}$$

$$\therefore U_2V\acute{}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = A_1 - U_2V\acute{}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & -1 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \neq \varphi$$

Seja A_2, $a_{43} = -1$

obter o produto $U_3V\acute{}_3$

$$U_3 = -\frac{1}{1}\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \qquad V\acute{}_3 = \begin{bmatrix} 0 & 0 & -1 & -1 \end{bmatrix}$$

$$U_3V\acute{}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$A_3 = A_2 - U_3V\acute{}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 0$$

Como $A_3 = \varphi$, então o processo está encerrado

$$B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}; \qquad C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$\text{Logo } A = BC \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 2 \\ 0 & 2 & 2 & 2 \\ 1 & 0 & 0 & -1 \end{bmatrix} = A$$

5) Obtenha a inversa de Moore-Penrose das matrizes abaixo, obtenha também uma inversa condicional e uma inversa de quadrado mínimos.

a) $\begin{pmatrix} 1 & \frac{1}{3} \\ \frac{-3}{2} & -3 \end{pmatrix}$

Seja em $A, a_{11} = 1$

$$U_1 = \frac{1}{1}\begin{bmatrix} 1 \\ \frac{-3}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{3}{2} \end{bmatrix}; \qquad V\acute{}_1 = \begin{bmatrix} 1 & \frac{1}{3} \end{bmatrix}$$

$$U_1V'_1 = \begin{bmatrix} 1 \\ -\frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{3} \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

$$A_1 = A - U_1V'_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{5}{2} \end{bmatrix} \neq \varphi$$

Seja em $A_1, a_{22} = -\frac{5}{2}$

$$U_2 = -\frac{2}{5} \begin{bmatrix} 0 \\ -\frac{5}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \qquad V'_2 = \begin{bmatrix} 0 & -\frac{5}{2} \end{bmatrix}$$

$$U_2V'_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & -\frac{5}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{5}{2} \end{bmatrix}$$

$$A_2 = A_1 - U_2V'_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \varphi$$

Então tem-se: $B = \begin{bmatrix} 1 & 0 \\ -\frac{3}{2} & 1 \end{bmatrix}$, transposta de $B; B' = \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 1 \end{bmatrix}$ e

$C = \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & -\frac{5}{2} \end{bmatrix}$, transposta de $C; C' = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & -\frac{5}{2} \end{bmatrix}$

$$A^+ = C'(CC')^{-1}(B'B)^{-1}B'$$

$$A^+ = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & -\frac{5}{2} \end{bmatrix} \left(\begin{bmatrix} 1 & \frac{1}{3} \\ 0 & -\frac{5}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & -\frac{5}{2} \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{3}{2} & 1 \end{bmatrix} \right)^{-1} \times$$

$$\times \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 1 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & -\frac{5}{2} \end{bmatrix} \left(\begin{bmatrix} \frac{10}{9} & -\frac{5}{6} \\ -\frac{5}{6} & \frac{25}{4} \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} \frac{13}{4} & -\frac{3}{2} \\ -\frac{3}{2} & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 1 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & -\frac{5}{2} \end{bmatrix} \begin{bmatrix} 1 & \frac{2}{15} \\ \frac{2}{15} & \frac{8}{45} \end{bmatrix} \begin{bmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & \frac{13}{4} \end{bmatrix} \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 1 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} \frac{6}{5} & \frac{2}{15} \\ -\frac{3}{5} & -\frac{2}{5} \end{bmatrix}$$

Inversa condicional A$^-$

$$\begin{pmatrix} 1 & \frac{1}{3} \\ \frac{-3}{2} & -3 \end{pmatrix}, \text{ rank: } 2$$

$$M = \begin{bmatrix} 1 & \frac{1}{3} \\ \frac{-3}{2} & -3 \end{bmatrix}$$

$$\left(M^{-1}\right)' = \begin{bmatrix} 1 & \frac{1}{3} \\ \frac{-3}{2} & -3 \end{bmatrix}^{-1\,'} = \begin{bmatrix} \frac{6}{5} & \frac{2}{15} \\ -\frac{3}{5} & -\frac{2}{5} \end{bmatrix}' = \begin{bmatrix} \frac{6}{5} & -\frac{3}{5} \\ \frac{2}{15} & -\frac{2}{5} \end{bmatrix}$$

$$\begin{bmatrix} \frac{6}{5} & -\frac{3}{5} \\ \frac{2}{15} & -\frac{2}{5} \end{bmatrix}' = \begin{bmatrix} \frac{6}{5} & \frac{2}{15} \\ -\frac{3}{5} & -\frac{2}{5} \end{bmatrix}$$

Portanto $\begin{bmatrix} \frac{6}{5} & \frac{2}{15} \\ -\frac{3}{5} & -\frac{2}{5} \end{bmatrix}$ é uma inversa condicional de $\begin{bmatrix} 1 & \frac{1}{3} \\ \frac{-3}{2} & -3 \end{bmatrix}$.

Inversa dos quadrados mínimos

$$\begin{bmatrix} 1 & \frac{1}{3} \\ \frac{-3}{2} & -3 \end{bmatrix} \qquad A^{l} = (A'A)^{-} A'$$

$A^{\prime} = \left(\begin{bmatrix} 1 & -\frac{3}{2} \\ \frac{1}{3} & -3 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{3} \\ \frac{-3}{2} & -3 \end{bmatrix} \right)^{-} \begin{bmatrix} 1 & -\frac{3}{2} \\ \frac{1}{3} & -3 \end{bmatrix}$

$A^{\prime} = \left(\begin{bmatrix} \frac{13}{4} & \frac{29}{6} \\ \frac{29}{6} & \frac{82}{9} \end{bmatrix} \right)^{-} \begin{bmatrix} 1 & -\frac{3}{2} \\ \frac{1}{3} & -3 \end{bmatrix}$

$A^{\prime} = \begin{bmatrix} \frac{328}{225} & -\frac{58}{75} \\ -\frac{58}{75} & \frac{13}{25} \end{bmatrix} \begin{bmatrix} 1 & -\frac{3}{2} \\ \frac{1}{3} & -3 \end{bmatrix}$

$A^{\prime} = \begin{bmatrix} \frac{6}{5} & \frac{2}{15} \\ -\frac{3}{5} & -\frac{2}{5} \end{bmatrix}$

b) $\begin{pmatrix} 1 & -1 & -2 \\ 0 & 3 & 0 \\ 3 & 4 & 0 \end{pmatrix}$

Seja em $A, a_{11} = 1$

$U_1 = \frac{1}{1} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}; \qquad V\acute{}_1 = \begin{bmatrix} 1 & -1 & -2 \end{bmatrix}$

$U_1 V\acute{}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 0 & 0 \\ 3 & -3 & -6 \end{bmatrix}$

$A_1 = A - U_1 V\acute{}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 7 & 6 \end{bmatrix}$

Seja em $A_1, a_{22} = 3$

$U_2 = \frac{1}{3} \begin{bmatrix} 0 \\ 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \frac{7}{3} \end{bmatrix}; \qquad V\acute{}_2 = \begin{bmatrix} 0 & 3 & 0 \end{bmatrix}$

$U_2 V\acute{}_2 = \begin{bmatrix} 0 \\ 1 \\ \frac{7}{3} \end{bmatrix} \begin{bmatrix} 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 7 & 0 \end{bmatrix}$

$A_2 = A_1 - U_2 V\acute{}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Seja em $A_2, a_{33} = 6$

$U_3 = \frac{1}{6} \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \qquad V\acute{}_3 = \begin{bmatrix} 0 & 0 & 6 \end{bmatrix}$

$U_3 V\acute{}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$A_3 = A_2 - U_3 V\acute{}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \varphi$

Então tem-se: $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & \frac{7}{3} & 1 \end{bmatrix}$, transposta de $B; B\acute{} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & \frac{7}{3} \\ 0 & 0 & 1 \end{bmatrix}$ e

$C=\begin{bmatrix}1&-1&-2\\0&3&0\\0&0&6\end{bmatrix}$ transposta de $C; C' = \begin{bmatrix}1&0&0\\-1&3&0\\-2&0&6\end{bmatrix}$

$A^{+}=C'(CC')^{-1}\left(B'B\right)^{-1}B'$

$$A^{+}=\begin{bmatrix}1&0&0\\-1&3&0\\-2&0&6\end{bmatrix}\left(\begin{bmatrix}1&-1&-2\\0&3&0\\0&0&6\end{bmatrix}\begin{bmatrix}1&0&0\\-1&3&0\\-2&0&6\end{bmatrix}\right)^{-1}\times$$
$$\times\left(\begin{bmatrix}1&0&3\\0&1&\frac{7}{3}\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\3&\frac{7}{3}&1\end{bmatrix}\right)^{-1}\begin{bmatrix}1&0&3\\0&1&\frac{7}{3}\\0&0&1\end{bmatrix}$$
$$A^{+}=\begin{bmatrix}1&0&0\\-1&3&0\\-2&0&6\end{bmatrix}\left(\begin{bmatrix}6&-3&-12\\-3&9&0\\-12&0&36\end{bmatrix}\right)^{-1}\left(\begin{bmatrix}10&7&3\\7&\frac{58}{9}&\frac{7}{3}\\3&\frac{7}{3}&1\end{bmatrix}\right)^{-1}\begin{bmatrix}1&0&3\\0&1&\frac{7}{3}\\0&0&1\end{bmatrix}$$

$$A^{+}=\begin{bmatrix}1&0&0\\-1&3&0\\-2&0&6\end{bmatrix}\begin{bmatrix}1&\frac{1}{3}&\frac{1}{3}\\\frac{1}{3}&\frac{2}{9}&\frac{1}{9}\\\frac{1}{3}&\frac{1}{9}&\frac{5}{36}\end{bmatrix}\begin{bmatrix}1&0&-3\\0&1&-\frac{7}{3}\\-3&-\frac{7}{3}&\frac{139}{9}\end{bmatrix}\begin{bmatrix}1&0&3\\0&1&\frac{7}{3}\\0&0&1\end{bmatrix}$$

$$A^{+}=\begin{bmatrix}0&-\frac{4}{9}&\frac{1}{3}\\0&\frac{1}{3}&0\\-\frac{1}{2}&-\frac{7}{18}&\frac{1}{6}\end{bmatrix}$$

Cálculo da inversa condicional de $\begin{pmatrix}1&-1&-2\\0&3&0\\3&4&0\end{pmatrix}$

$$\left(\begin{bmatrix}1&-1&-2\\0&3&0\\3&4&0\end{bmatrix}^{-1}\right)'=\begin{bmatrix}0&-\frac{4}{9}&\frac{1}{3}\\0&\frac{1}{3}&0\\-\frac{1}{2}&-\frac{7}{18}&\frac{1}{6}\end{bmatrix}'=\begin{bmatrix}0&0&-\frac{1}{2}\\-\frac{4}{9}&\frac{1}{3}&-\frac{7}{18}\\\frac{1}{3}&0&\frac{1}{6}\end{bmatrix},$$

Portanto $\begin{bmatrix}0&-\frac{4}{9}&\frac{1}{3}\\0&\frac{1}{3}&0\\-\frac{1}{2}&-\frac{7}{18}&\frac{1}{6}\end{bmatrix}$ é a inversa condicional de $\begin{pmatrix}1&-1&-2\\0&3&0\\3&4&0\end{pmatrix}$

Cálculo da inversa dos quadrados mínimosde $\begin{pmatrix}1&-1&-2\\0&3&0\\3&4&0\end{pmatrix}$, transpose:

$A^{\shortmid}=(A'A)^{-}A'$

$$A^{\shortmid}=\left(\begin{bmatrix}1&0&3\\-1&3&4\\-2&0&0\end{bmatrix}\begin{bmatrix}1&-1&-2\\0&3&0\\3&4&0\end{bmatrix}\right)^{-}\begin{bmatrix}1&0&3\\-1&3&4\\-2&0&0\end{bmatrix}$$

$A^{\prime} = \begin{bmatrix} \frac{25}{81} & -\frac{4}{27} & \frac{37}{162} \\ -\frac{4}{27} & \frac{1}{9} & -\frac{7}{54} \\ \frac{37}{162} & -\frac{7}{54} & \frac{139}{324} \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ -1 & 3 & 4 \\ -2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{4}{9} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 \\ -\frac{1}{2} & -\frac{7}{18} & \frac{1}{6} \end{bmatrix}$ é a inversa dos quadrados mínimos

c) $\begin{pmatrix} 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix}$

Seja em $A, a_{12} = 2$
$U_1 = \frac{1}{2} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad V_1' = \begin{bmatrix} 0 & 2 & 1 & 0 \end{bmatrix}$
$U_1 V_1' = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{bmatrix}$
$A_1 = A - U_1 V_1' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \varphi$
Então tem-se: $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, transposta de $B; B' = \begin{bmatrix} 1 & 1 \end{bmatrix}$ e

$C = \begin{bmatrix} 0 & 2 & 1 & 0 \end{bmatrix}$ transposta de $C; C' = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$

$A^{+} = C'(CC')^{-1} (B'B)^{-1} B'$
$A^{+} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} \left(\begin{bmatrix} 0 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 \end{bmatrix}$

$A^{+} = \begin{bmatrix} 0 & 0 \\ \frac{1}{5} & \frac{1}{5} \\ \frac{1}{10} & \frac{1}{10} \\ 0 & 0 \end{bmatrix}$
Inversa condicional de $\begin{pmatrix} 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix}$ é dada por:
$M = \left([2]^{-1} \right)' = \frac{1}{2}$
$\begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ logo $\begin{pmatrix} 0 & 0 \\ \frac{1}{2} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ é uma da inversa condicional de $\begin{pmatrix} 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix}$

A inversa de quadrados mínimos é dada por:
$\begin{pmatrix} 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 8 & 4 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, rank: 1

$A^{\text{|}} = (A'A)^{-} A'$

$$A^{\text{|}} = \left(\begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{bmatrix} \right)^{-} \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^{\text{|}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^{\text{|}} = \begin{bmatrix} 0 & 0 \\ \frac{1}{4} & \frac{1}{4} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

6) Através do escalonamento de matrizes obtenha valores para a e b, se estes existirem, tal que o sistema seja inconsistentes. Neste caso obtenha uma solução aproximadamente utilizando a inversa de Moore_Penrose.

$$\begin{cases} x_1 - x_2 + x_3 = 1 \\ x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + x_2 + ax_3 = b \end{cases} \Longrightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & : & 1 \\ 1 & 2 & -1 & : & 1 \\ 2 & 1 & a & : & b \end{bmatrix} \begin{matrix} \\ -L_1 + L_2 \\ -2L_1 + L_3 \end{matrix} \Longrightarrow \begin{bmatrix} 1 & -1 & 1 & : & 1 \\ 0 & 3 & -2 & : & 0 \\ 0 & 3 & a-2 & : & b-2 \end{bmatrix} \begin{matrix} \\ \\ -L_2 + L_3 \end{matrix} \Longrightarrow$$

$$\Longrightarrow \begin{bmatrix} 1 & -1 & 1 & : & 1 \\ 0 & 3 & -2 & : & 0 \\ 0 & 0 & a & : & b-2 \end{bmatrix} \Longrightarrow \text{para que o sistema seje inconsistente}$$

temos $a = b$ e qualquer valor que satisfaça esta condição, por exemplo $a = 1$ e $b = 1$

tem-se $\begin{bmatrix} 1 & -1 & 1 & : & 1 \\ 0 & 3 & -2 & : & 0 \\ 0 & 0 & 1 & : & -1 \end{bmatrix} \Longrightarrow r(A) = 2$ e $r(A : y) = 3$

Como $r(A) = 2 < r(A : y) = 3$, o sistema é inconsistent; logo nao existe combinação linear das colunasde A que produza y

$x_+ = A^+ y$ é a solução aproximada para $Ax = y$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x^1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$x_+ = \begin{bmatrix} 1 & \frac{2}{3} & -\frac{1}{3} \\ -1 & -\frac{1}{3} & \frac{2}{3} \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} \\ -\frac{2}{3} \\ -1 \end{bmatrix}$$

norma do erro

$$e\left(x^1\right) = y - Ax^1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \\ -8 \end{bmatrix}$$

$\|e(x)\|^2 = (-1)^2 + (-5)^2 + (-8)^2 = 90$

$$e(x^*) = y - Ax^* = \left(\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{4}{3} \\ -\frac{2}{3} \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\|e(x^*)\|^2 = 0$

$\|(x)\|^2 = 90 > \|(x^*)\|^2 = 0$

Norma da solução

$\|(x)\|^2 = 3^2 + 2^2 + 1^2 = 14$, $\|(x^*)\|^2 = \left(\frac{4}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + (-1)^2 = \frac{29}{9}$

Existe vetores x menor que $\frac{29}{9}$.

Cálculo auxiliar de A^+ no exercício 6.

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$U1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \qquad V1 = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}$$

$$U1V1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 2 \end{bmatrix}$$

$$A1 = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 3 & -1 \end{bmatrix}$$

$$U2 = \frac{1}{3} \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad V2 = \begin{bmatrix} 0 & 3 & -2 \end{bmatrix}$$

$$U2V2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{bmatrix}$$

$$\text{A2=} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 3 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{U3=} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \text{V3=} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$U3V3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \varphi$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}, \text{transpose: } \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\text{transpose: } \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A^+ = C'(CC')^{-1}\,(B'B)^{-1}\,B'$$

$$A^+ = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 1 & -2 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 1 & -2 & 1 \end{bmatrix} \right)^{-1} \times$$

$$\times \left(\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 1 & \frac{2}{3} & -\frac{1}{3} \\ -1 & -\frac{1}{3} & \frac{2}{3} \\ -1 & -1 & 1 \end{bmatrix}$$